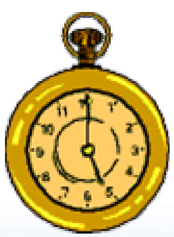
AccuBeat Ltd.
Accurate Frequency & Time

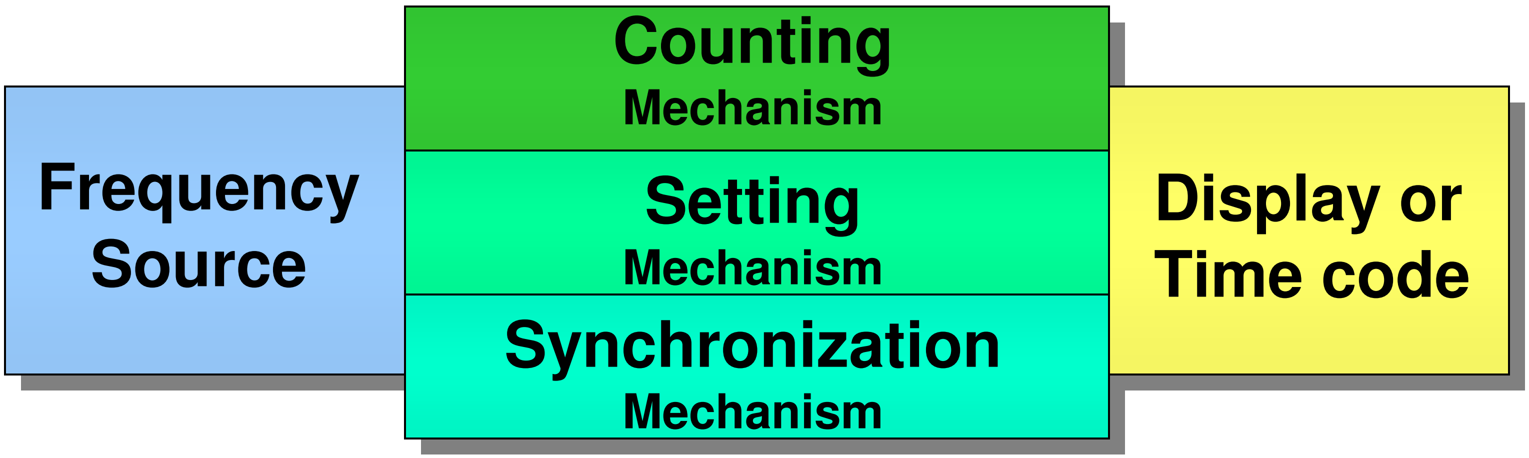
Time and Frequency Fundamentals



What is Time

- "What, then, is time? If no one asks me, I know; If I wish to explain to him who asks, I know not." --- Saint Augustine, circa 400 A.D.
- The question, both a philosophical and a scientific one, has no entirely satisfactory answer. "Time is what a clock measures." "It defines the temporal order of events." "It is an element in the four-dimensional geometry of space-time." "It is nature's way of making sure that everything doesn't happen at once."
- Why are there "arrows" of time? The arrows are: entropy, electromagnetic waves, expansion of the universe, k-meson decay, and psychological. Does time have a beginning and an end? (Big bang; no more "events", eventually.) See. e.g., Time's Arrows, by Richard Morris, Simon & Schuster, NY, 1985.
- The unit of time, the second, is one of the seven base units in the International System of Units (SI units). Since time is the quantity that can be measured with the highest accuracy, it plays a central role in metrology.

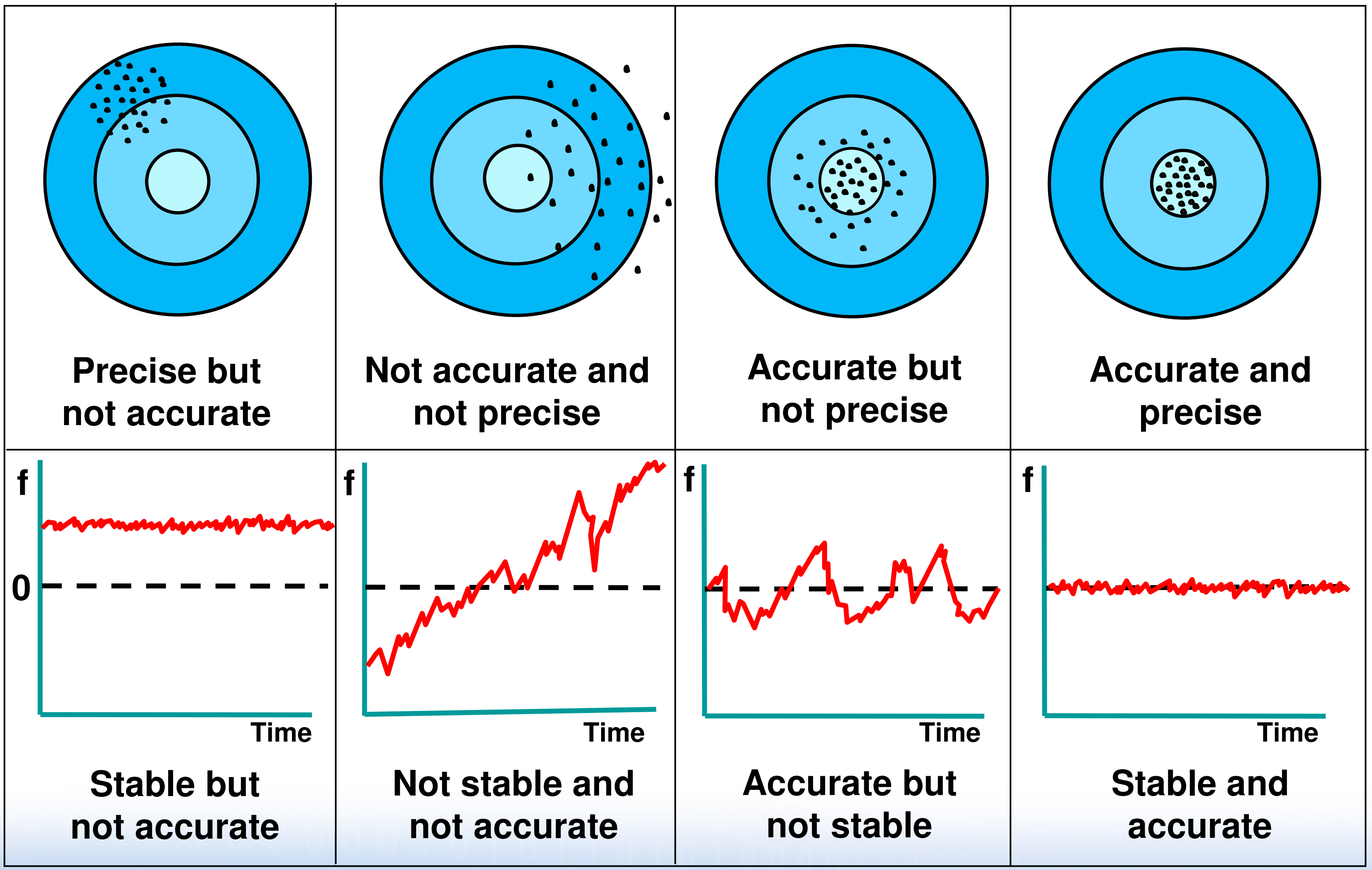
Typical Clock System



$$t = t_0 + \Sigma\Delta\tau$$

Where t is the time output, t_0 is the initial setting, and $\Delta\tau$ is the time interval being counted.

Accuracy, Precision & Stability



The Second

- The SI unit of time is the second (symbol s).
- The second was defined, by international agreement, in October, 1967, at the XIII General Conference of Weights and Measures.
- **The second is "the duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium atom 133."**
- Prior to 1967, the unit of time was based on astronomical observations; the second was defined in terms of ephemeris time, i.e., as "1/31,556,925.9747 of the tropical year..."
- The unit of frequency is defined as the hertz (symbol Hz). One hertz equals the repetitive occurrence of one "event" per second.

Progress in Timekeeping

Time Period	Clock/Milestone	Accuracy Per Day
4th millennium B.C.	Day & night divided into 12 equal hours	
Up to 1280 A.D.	Sundials, water clocks (clepsydrae)	~1 h
~1280 A.D.	Mechanical clock invented- assembly time for prayer was first regular use	~30 to 60 min
14th century	Invention of the escapement; clock making becomes a major industry	~15 to 30 min
~1345	Hour divided into minutes and seconds	
15th century	Clock time used to regulate people's lives (work hours)	~2 min
16th century	Time's impact on science becomes significant (Galileo times physical events, e.g., free-fall)	~1 min
1656	First pendulum clock (Huygens)	~100 s
18th century	Temperature compensated pendulum clocks	1 to 10 s
19th century	Electrically driven free-pendulum clocks	10^{-2} to 10^{-1} s
~1910 to 1920	Wrist watches become widely available	
1920 to 1934	Electrically driven tuning forks	10^{-3} to 10^{-2} s
1921 to present	Quartz crystal clocks (and watches. Since ~1971)	10^{-5} to 10^{-1} s
1949 to present	Atomic clocks	10^{-9} to 10^{-4} s

Time & Frequency Definitions

***Output Signal of a Precision Oscillator*:**

$$V(t) = [V_o + \epsilon(t)] \sin [2 \pi f_o t + \Phi(t)]$$

***Frequency*:** $f(t) = f_o + (d\Phi/dt) / 2\pi$

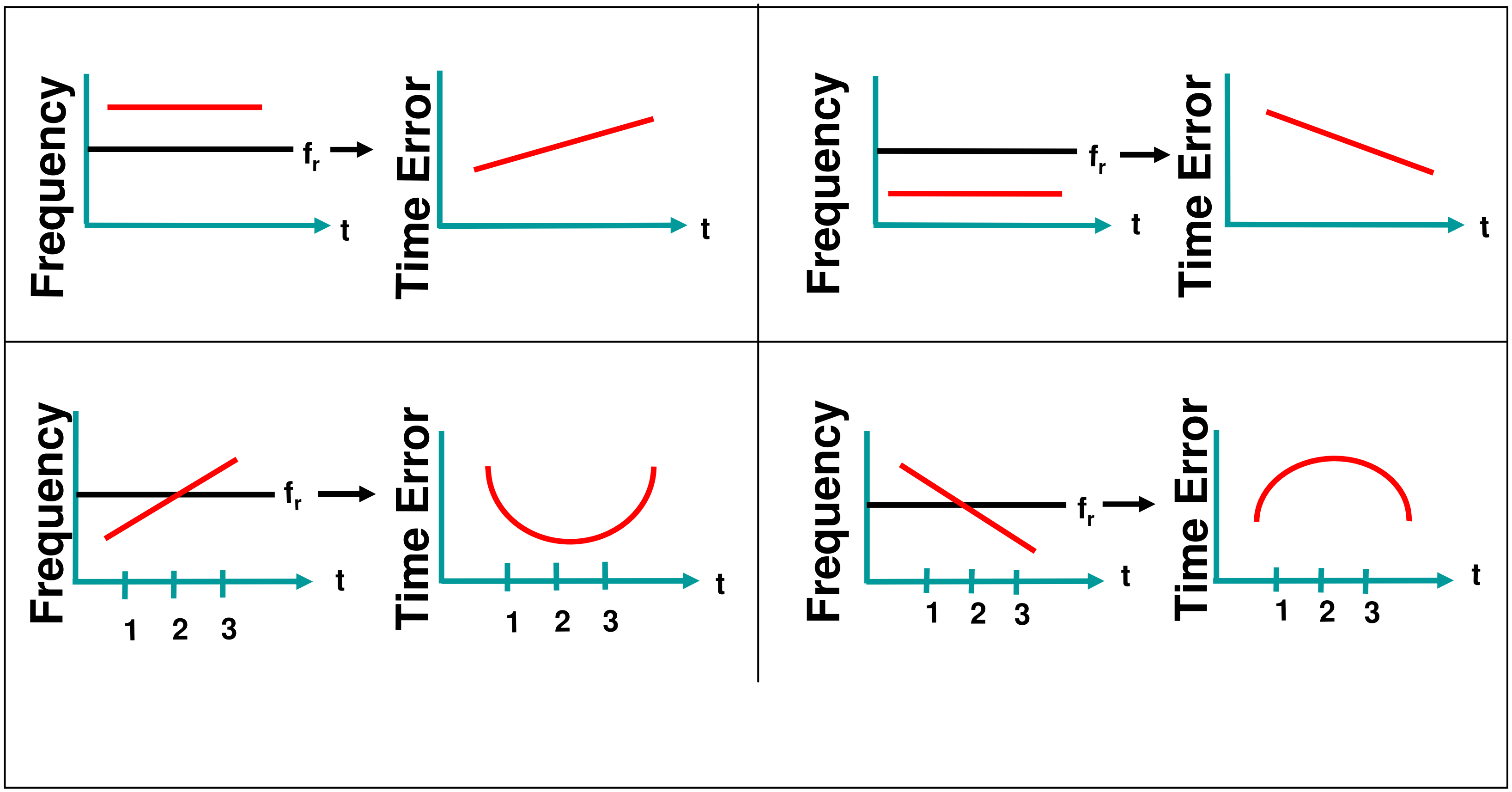
***Frequency Deviation from Nominal*:**

$$\Delta f = f(t) - f_o = (d\Phi/dt) / 2\pi$$

***Fractional Frequency Deviation*:** $y = \Delta f / f_o$

COURTESY OF DR. JOHN VIG

Frequency Error vs. Time Error



f_r = reference (i.e., the “correct”) frequency

Frequency & Time Errors

$$T - T_O = \int_0^t y(t)\, dt$$

Time Error = Integral of Fractional Frequency Error

$$y(t) = y_o + a\, t + e(t)$$

$$T - T_O = y_o\, t + (1/2)\, a\, t^2 + \int e(t)\, dt$$

$y = df / f$ a = linear frequency drift

y_o = initial frequency error $e(t)$ = frequency noise

Frequency & Time Errors

$$(T-T_O)/t = (1/t) \int_0^t y(t)\, dt = < y(t) >$$

$$dT / t = < df / f >$$

$T\text{-}T_O = dT$ = time error $\quad$ df = frequency error

t = elapsed time $\quad$ f = frequency

< > = average

<u>Example</u>:

The average frequency error of clock with time drift of 1ms per 1 month is:

$$df / f = dT / t = 1E\text{-}3\ s / 30 \times 24 \times 3600sec = 3.85E\text{-}10$$

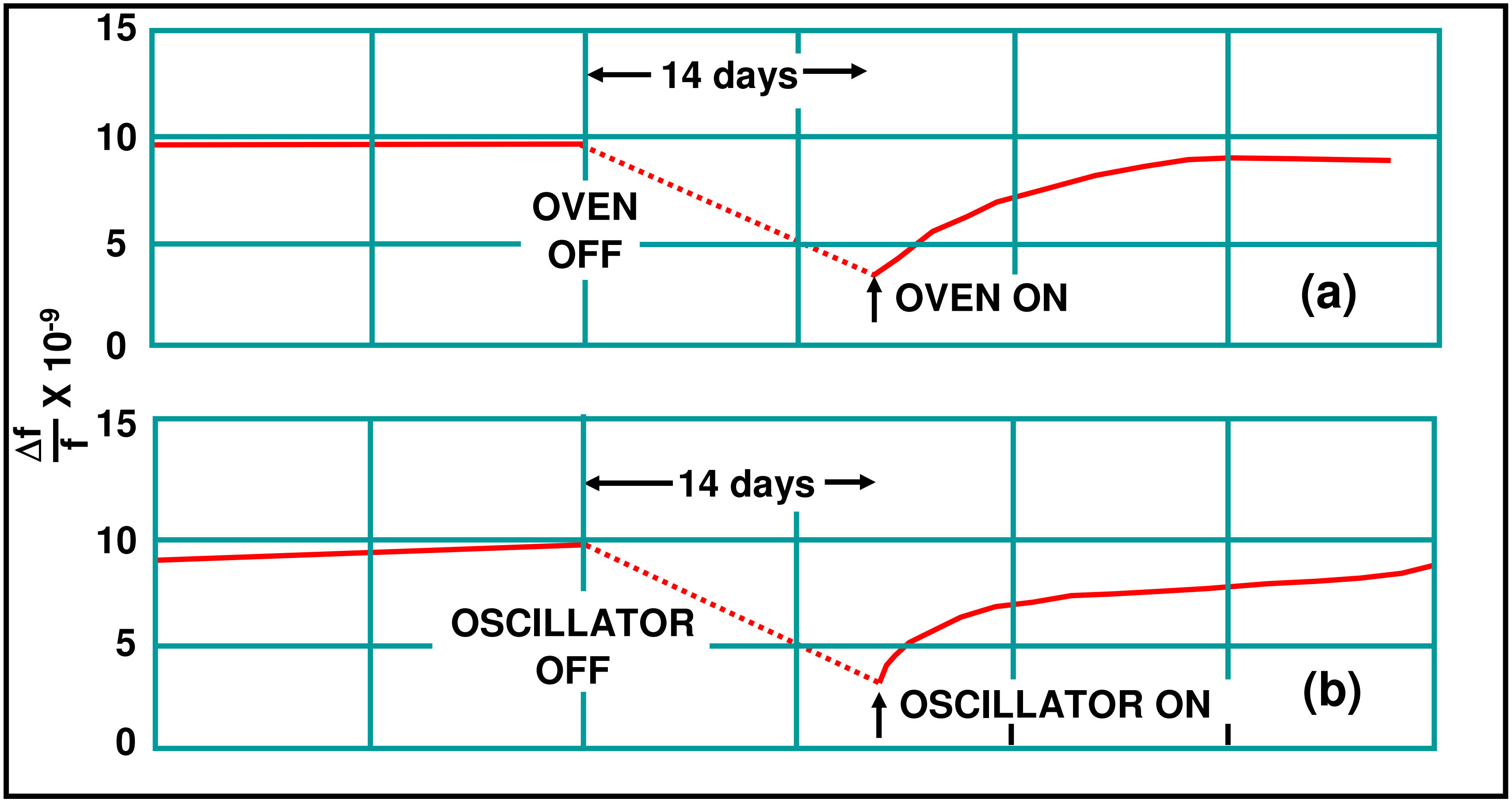
Frequency Terms

Warm-Up: *Time it takes for an Oscillator to reach a specified precision (compared to the frequency after stabilization).*

Retrace: *Turn-off the Oscillator for long time (say 1 day).*
Turn-on for sometime (say 1 hour).
Measure the frequency and compare to the frequency before the turn-off .

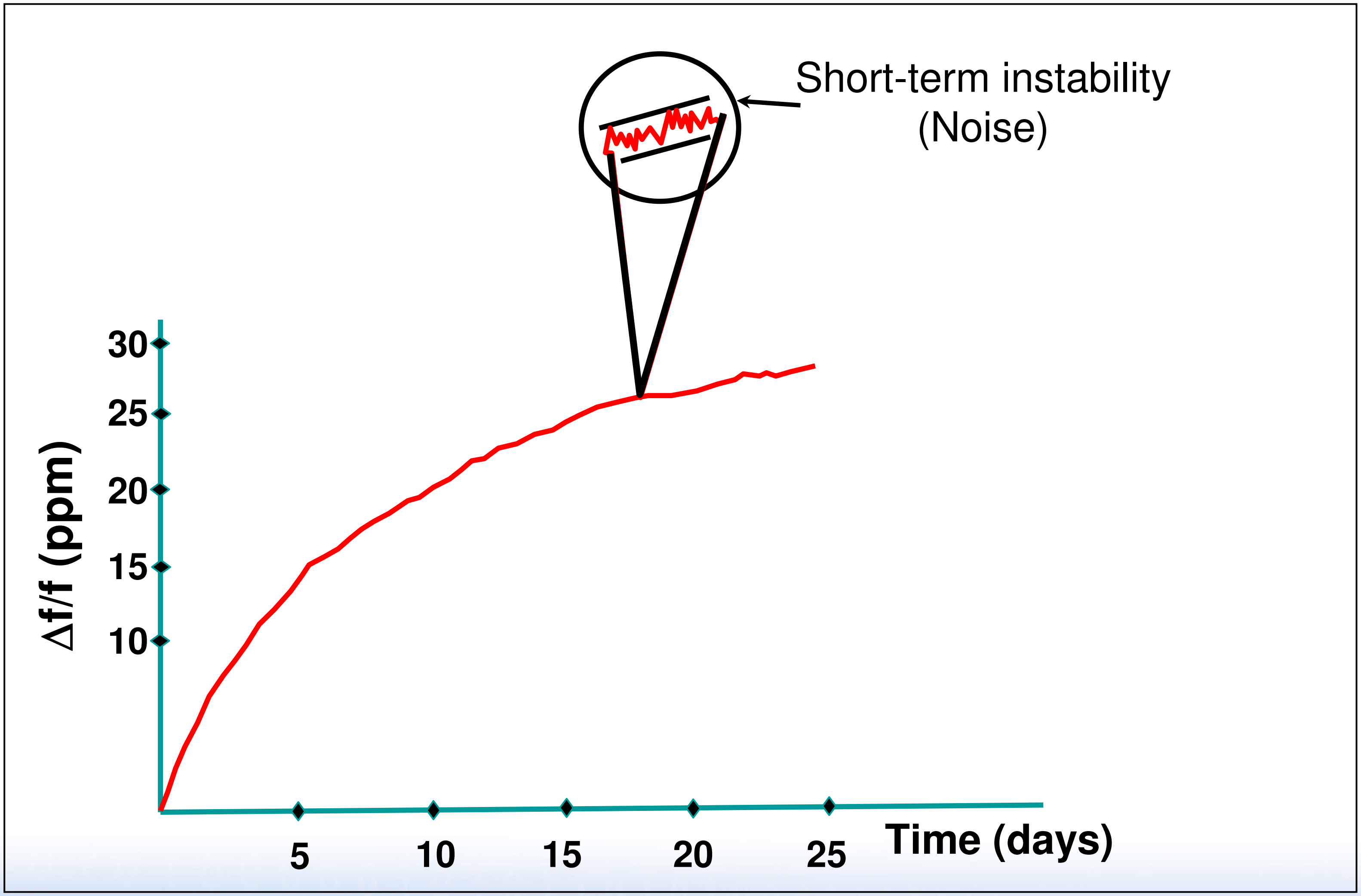
Aging: *The change in frequency of a Precision Oscillator in the course of time due to internal causes (not external or environmental)*

OCXO Retrace



In (a), the oscillator was kept on continuously while the oven was cycled off and on. In (b), the oven was kept on continuously while the oscillator was cycled off and on.

Aging and Short-Term Stability



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Allan Deviation

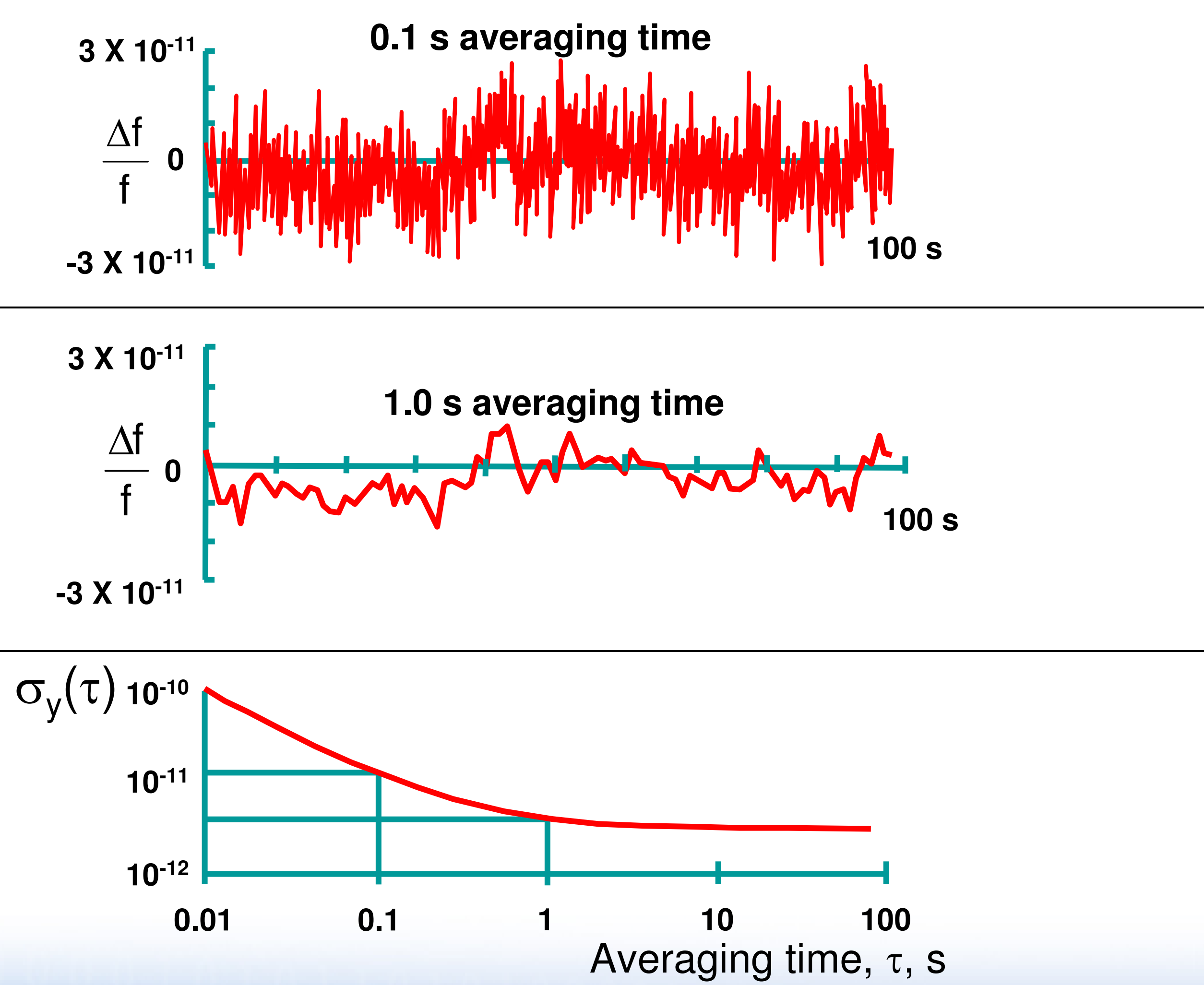
Also called **two-sample deviation,** or square-root of the "**Allan variance,**" it is the standard method of describing the short term stability of oscillators in the time domain. It is denoted by $\sigma_y(\tau)$,

where $$\sigma_y^2(\tau) = \frac{1}{2} < (y_{k+1} - y_k)^2 > .$$

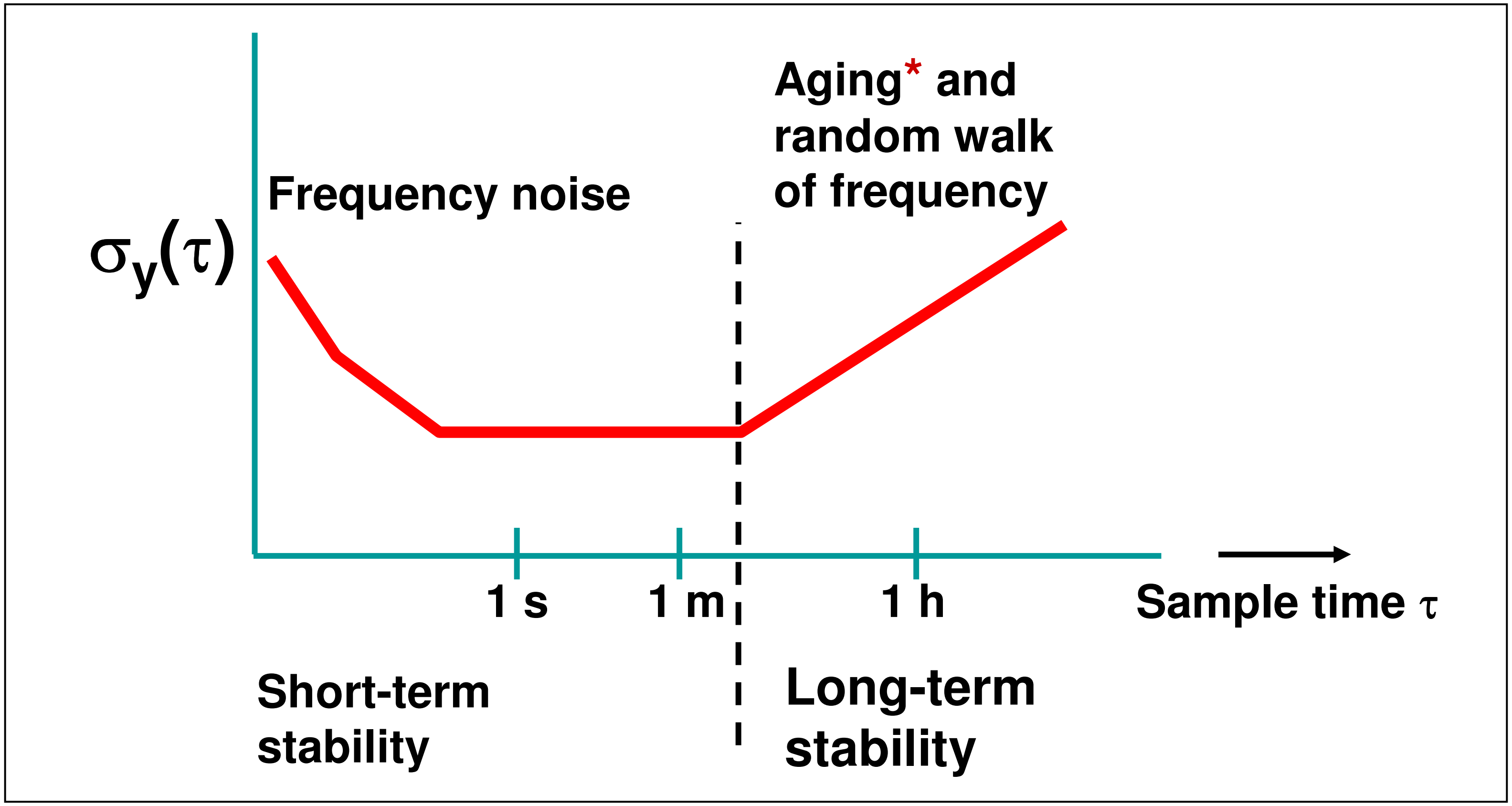
The fractional frequencies, $y = \frac{\Delta f}{f}$ are measured over a time interval, τ; $(y_{k+1} - y_k)$ are the differences between pairs of successive measurements of y, and, ideally, < > denotes a time average of an infinite number of $(y_{k+1} - y_k)^2$. A good estimate can be obtained by a limited number, m, of measurements ($m \geq 100$). $\sigma_y(\tau)$ generally denotes $\sqrt{\sigma_y^2(\tau, m)}$, i.e.,

$$\sigma_y^2(\tau) = \sigma_y^2(\tau, m) = \frac{1}{m}\sum_{j=1}^{m}\frac{1}{2}\left(y_{k+1} - y_k\right)_j^2$$

Frequency Noise and $\sigma y(\tau)$



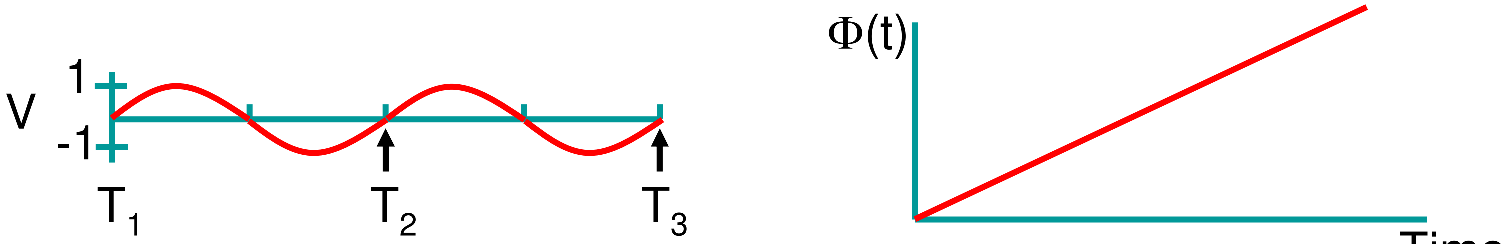
Time Domain Stability



*For $\sigma_y(\tau)$ to be a proper measure of random frequency fluctuations, aging must be properly subtracted from the data at long τ's.

Short Term Instability (Noise)

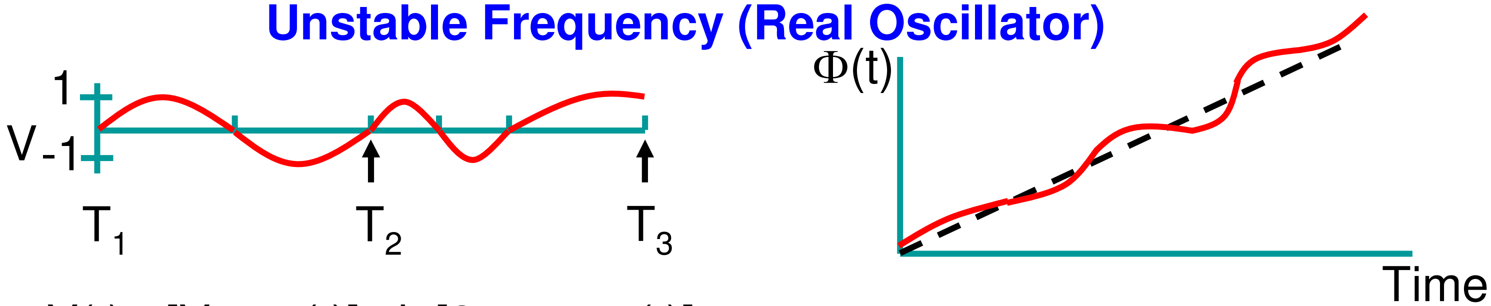
Stable Frequency (Ideal Oscillator)



$V(t) = V_0 \sin(2\pi\nu_0 t)$

$\Phi(t) = 2\pi\nu_0 t$

Unstable Frequency (Real Oscillator)



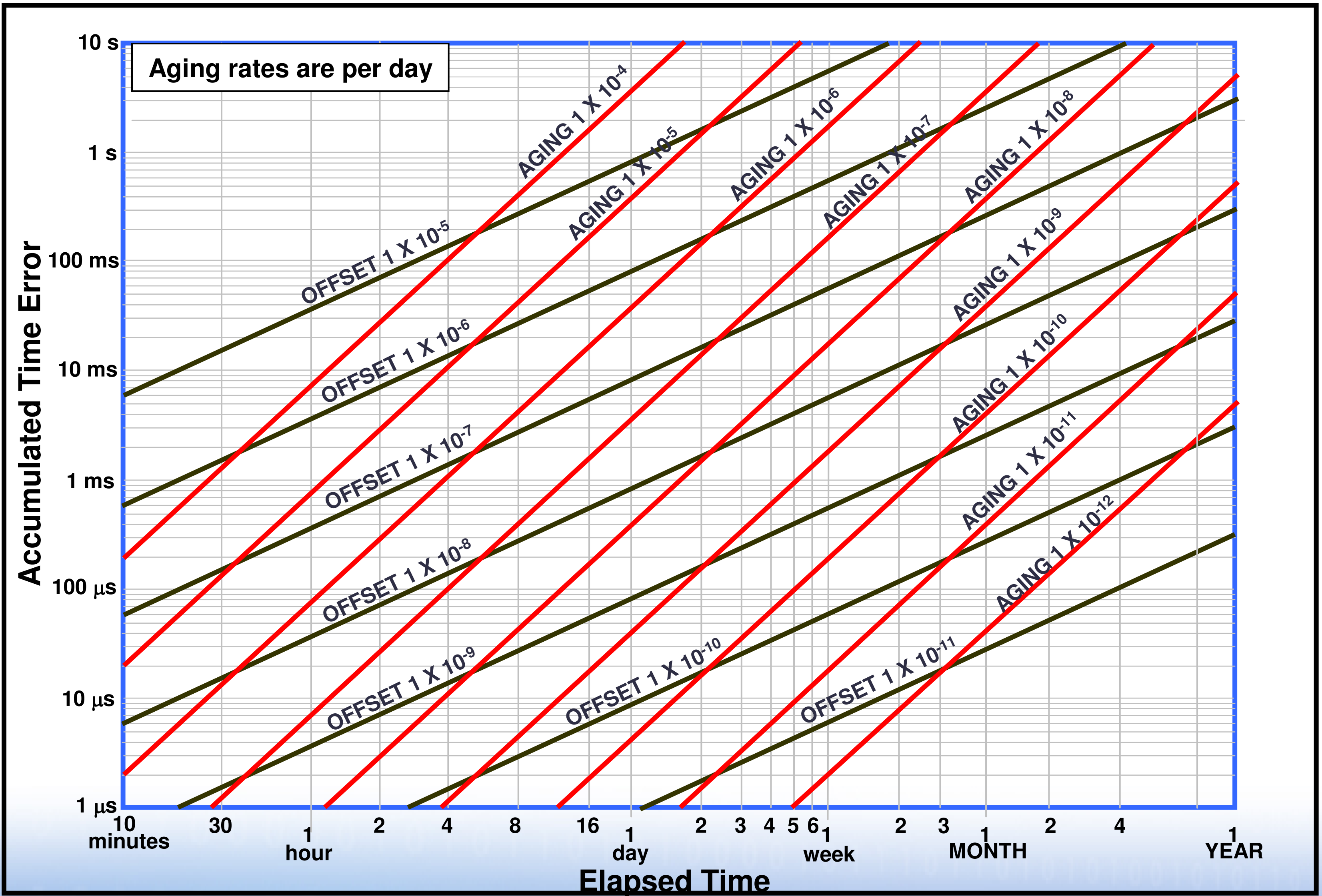
$V(t) = [V_0 + \varepsilon(t)] \sin[2\pi\nu_0 t + \phi(t)]$

$\Phi(t) = 2\pi\nu_0 t + \phi(t)$

Instantaneous frequency, $\nu(t) = \frac{1}{2\pi}\frac{d\Phi(t)}{dt} = \nu_0 + \frac{1}{2\pi}\frac{d\phi(t)}{dt}$

$V(t)$ = Oscillator output voltage, V_0 = Nominal peak voltage amplitude
$\varepsilon(t)$ = Amplitude noise, ν_0 = Nominal (or "carrier") frequency
$\Phi(t)$ = Instantaneous phase, and $\phi(t)$ = Deviation of phase from nominal (i.e., the ideal)

Time Error vs. Elapsed Time



Short-Term Stability Measures

Measure	Symbol
Two-sample deviation, also called "Allan deviation"	$\sigma_y(\tau)$*
Spectral density of phase deviations	$S_\phi(f)$
Spectral density of fractional frequency deviations	$S_y(f)$
Phase noise	$\mathcal{L}(f)$*
* Most frequently found on oscillator specification sheets	

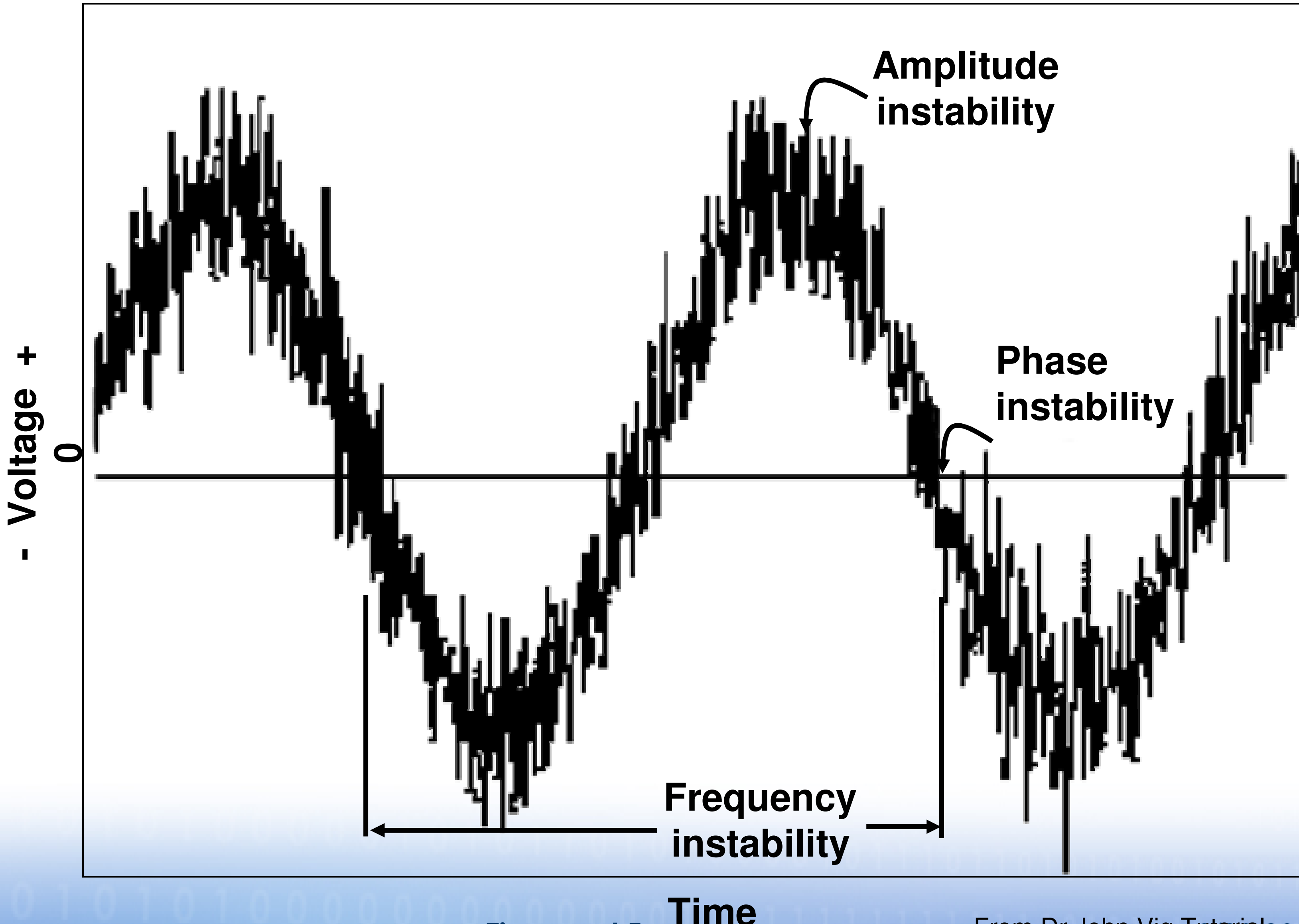
$f^2 S_\phi(f) = \nu^2 S_y(f)$; $\mathcal{L}(f) \equiv ½ [S_\phi(f)]$ (per IEEE Std. 1139),

and

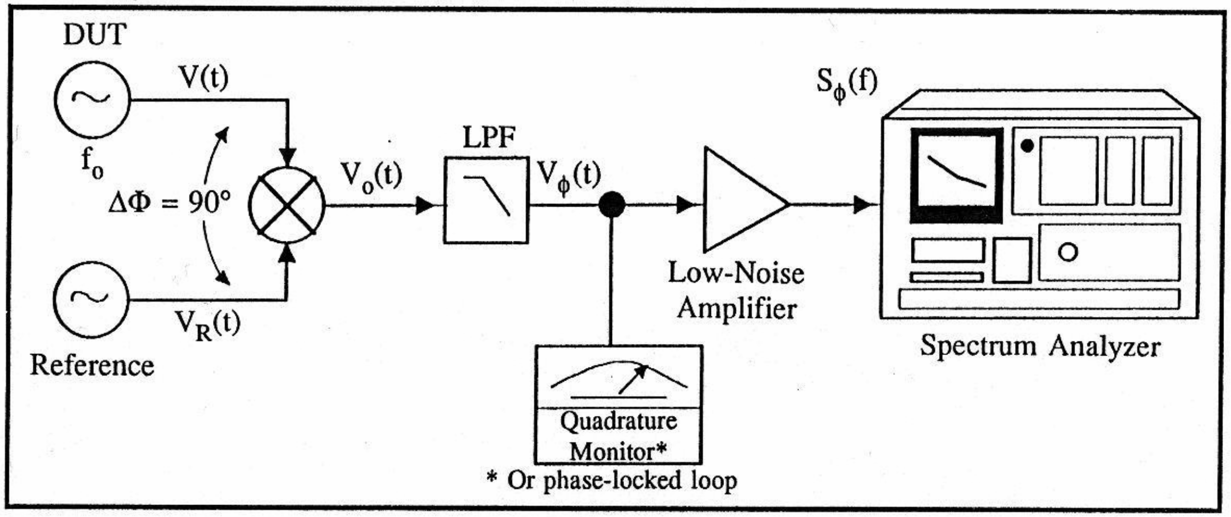
$$\sigma_y^2(\tau) = \frac{2}{(\pi\nu\tau)^2} \int_0^\infty S_\phi(f) \sin^4(\pi f \tau) df$$

Where τ = averaging time, ν = carrier frequency, and f = offset or Fourier frequency, or "frequency from the carrier".

Instantaneous Output Voltage of an Oscillator



Phase Detector



The device under test (DUT) and a reference source, at the same frequency and in phase quadrature (i.e., 90° out of phase), are input to a double-balanced mixer. Then,

$V_O(t) = V(t)\ V_R(t) = K \cos[\phi(t) - \phi_R(t) + \pi/2] + K \cos[2\pi(\nu + \nu_R)t + \ldots]$.

The low-pass filter (LPF) eliminates the second cosine term. Then,

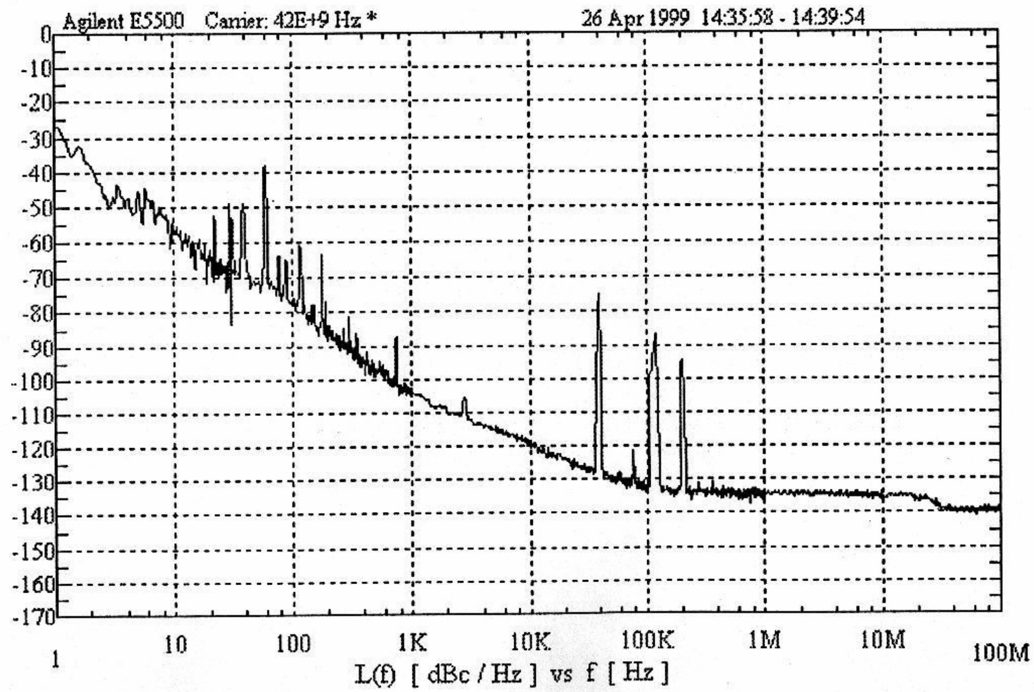
for $\phi_R(t) << \phi(t) << \pi/2$, $\quad \mathbf{V_\phi(t) = K\phi(t)}$,

i.e., the phase detector converts phase fluctuations to voltage fluctuations.

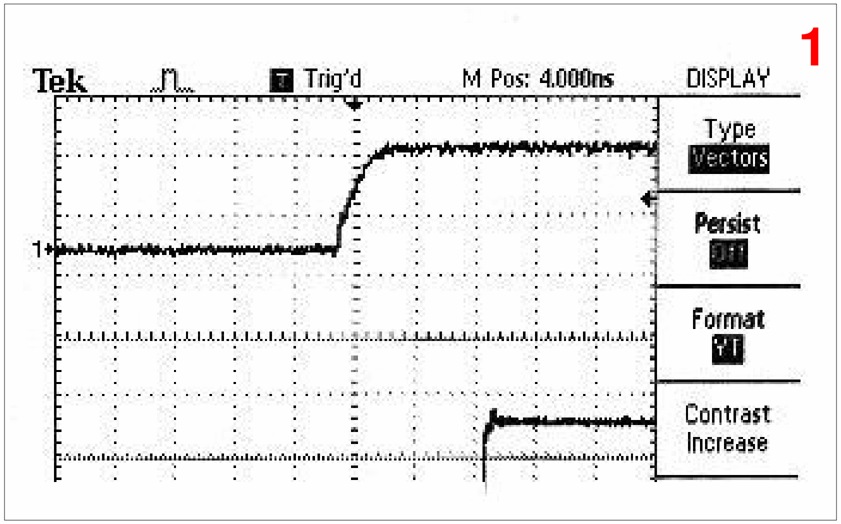
42 GHz Reference Source

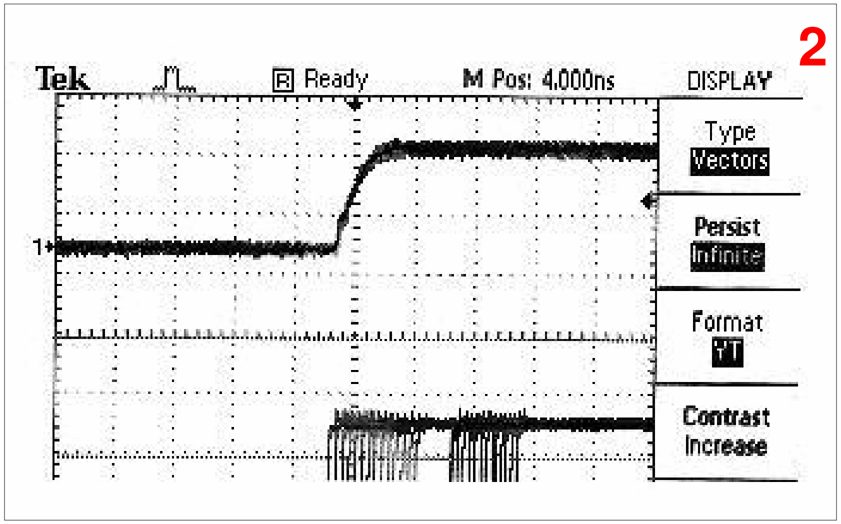
Type	Start(Hz)	Stop(Hz)	Value	Jitter	
Sphi(f)	10E +3	80E +6	1.73E-3 Rad	6.5 fsec.	275E-6 ui

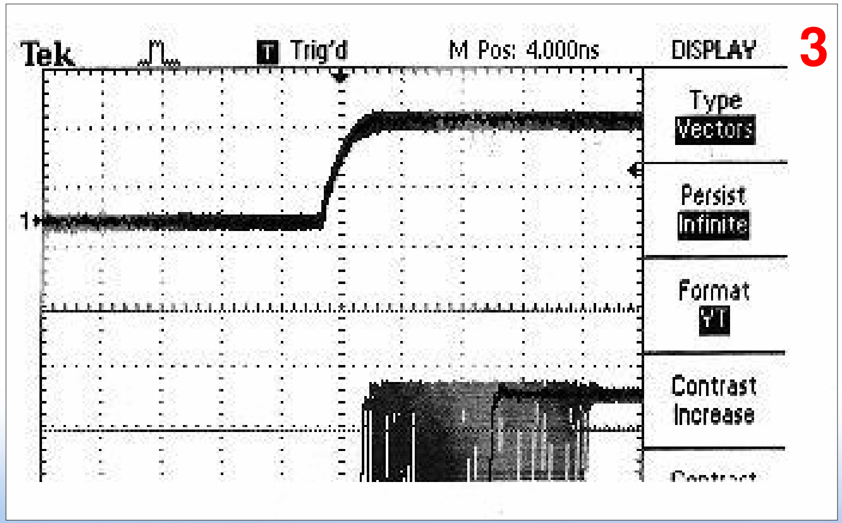
42 GHz Reference Source



Time Jitter


1
Tek
Trig'd
M Pos: 4.000ns
DISPLAY
Type
Vectors
Persist
Off
Format
YT
Contrast
Increase


2
Tek
Ready
M Pos: 4.000ns
DISPLAY
Type
Vectors
Persist
Infinite
Format
YT
Contrast
Increase


3
Tek
Trig'd
M Pos: 4.000ns
DISPLAY
Type
Vectors
Persist
Infinite
Format
YT
Contrast
Increase

Oscillator Acronyms

- XO .... Crystal Oscillator
- VCXO .... Voltage Controlled Crystal Oscillator
- TCXO .... Temperature Compensated Crystal Oscillator
- MCXO .... Microcomputer Compensated Crystal Oscillator
- OCXO .... Oven Controlled Crystal Oscillator
- Rb .... Rubidium Frequency Standard
- Cs .... Cesium Frequency Standard
- H Maser ..... Hydrogen Maser Frequency Standard

Time Code

- The need for Time Codes arose in the early missile and space programs.
- Time Codes are used to correlate with time test data recorded on magnetic tape.
- Time Codes are also transmitted on telemetry channels.
- Applications:

➢Missile	➢Communication
➢Space, Satellites	➢Commercial Data Acquisition
➢Flight Mission	➢Electrical Utilities – record facts

IRIG Serial Time Code

- Information is Pulsed Width Coded.
- The information is coded in Binary Code Decimal or Binary fashions.
- Time information includes Seconds, Minutes, Hours and Day of the year.
- Various formats designated A,B,….H differ by their Time Frame and BIT Rate.
- Most common is "IRIG B" with 1 second time frame and 100pps BIT Rate.
- In most cases the codes are Amlitude Modulated. For IRIG B the carrier is 1KHz and "1" = 5 large cycles "0" = 2 large cycles

Characteristics of Common Time Code Formats

CODE TYPE	BCD BITS	TIME INDICATOR	CODE FRAME LENGTH	CODE SCAN RATES	CODE CARRIER FREQUENCY	NOTE
IRIG A	34	Days, hours, minutes, seconds, 0.1 sec.	0.1 sec.	1000 pps	10 KHz	* 17 bit binary indicates time of day. Up to 27 control bits can be added.
IRIG B	30	Days, hours, minutes, seconds	1 sec.	100 pps	1 KHz	* 17 bit binary indicates time of day. Up to 27 control bits can be added.
IRIG D	16	Days, hours	1 hour	1 ppm	100 Hz or 1 KHz	UP to 9 control bits can be added.
IRIG E	26	Days, hours, minutes, seconds	10 sec.	10 pps	100 Hz or 1 KHz	UP to 27 control bits can be added.
IRIG G	38	Days, hours, minutes, seconds, 0.1 sec., 0.01 sec.	0.01 sec.	10,000 pps	100 KHz	UP to 36 control bits can be added.
IRIG H	23	Days, hours, minutes	1 min.	1 pps	100 Hz or 1KHz	UP to 9 control bits can be added.
NASA 36 bit	36	Days, hours, minutes, seconds	1 sec.	100 pps	1 KHz	UP to 4 control bits can be added.

IRIG B – Time Code

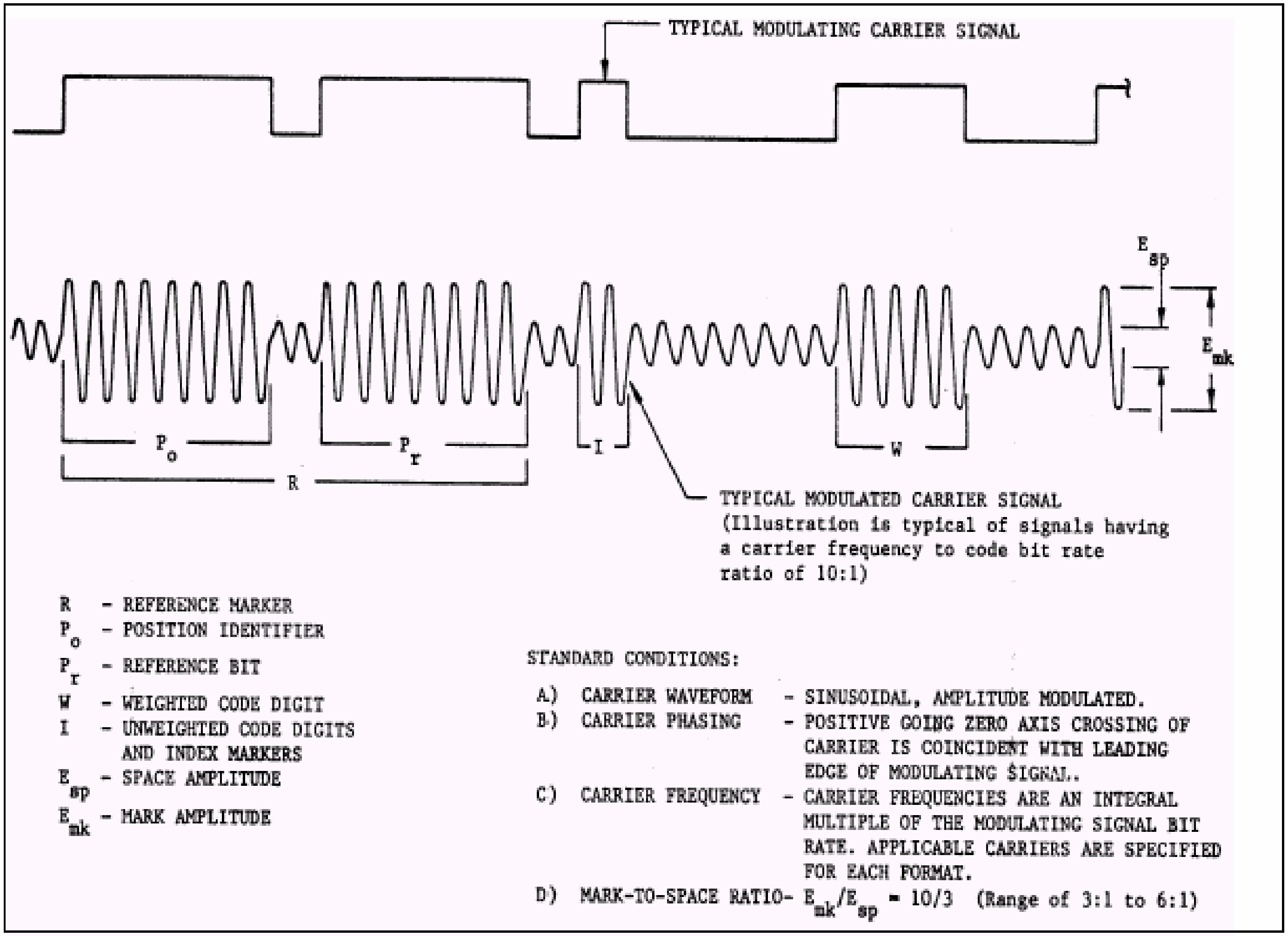


Figure 1. Typical modulated carrier signal.

IRIG B – Time Code

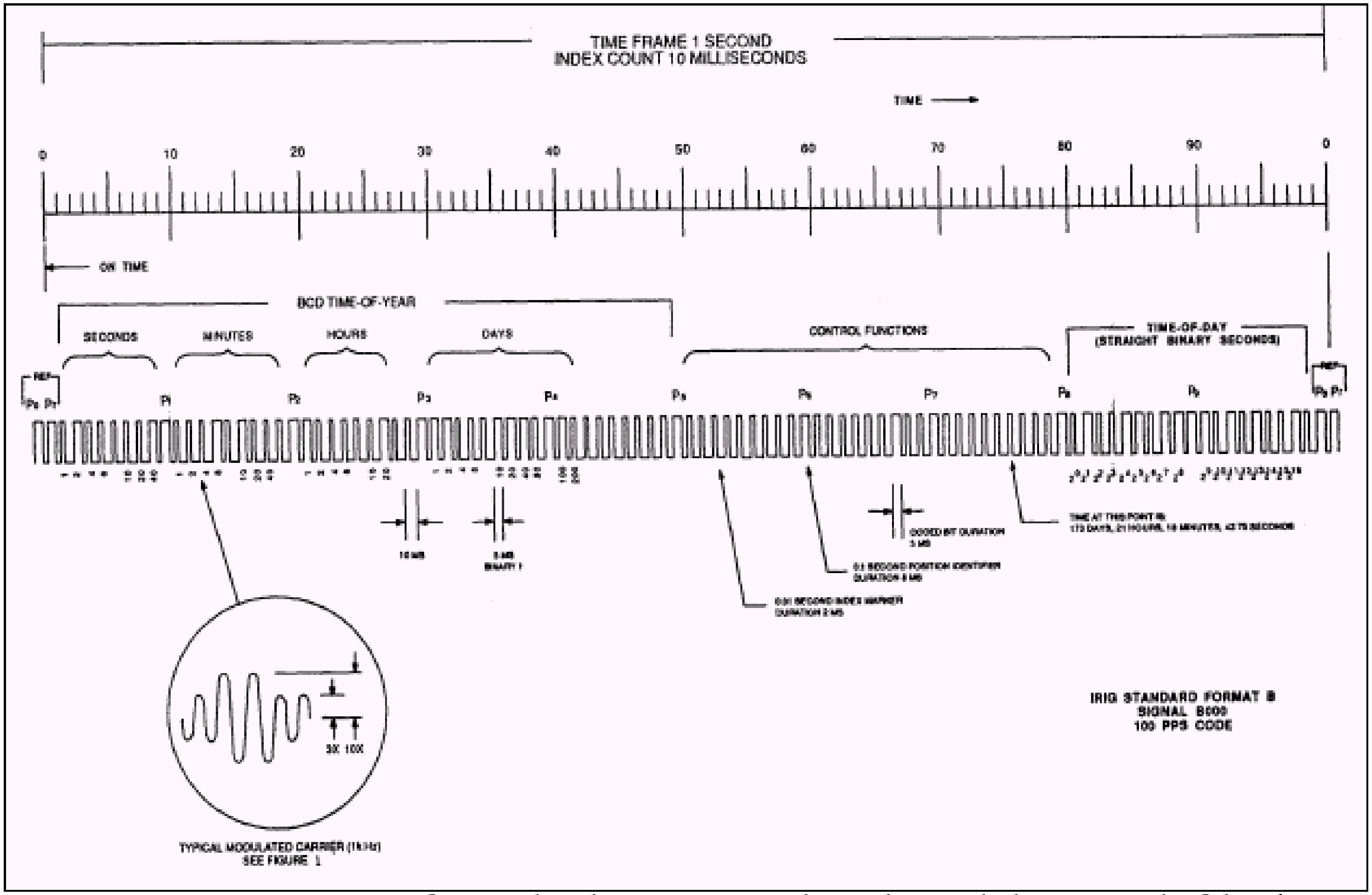


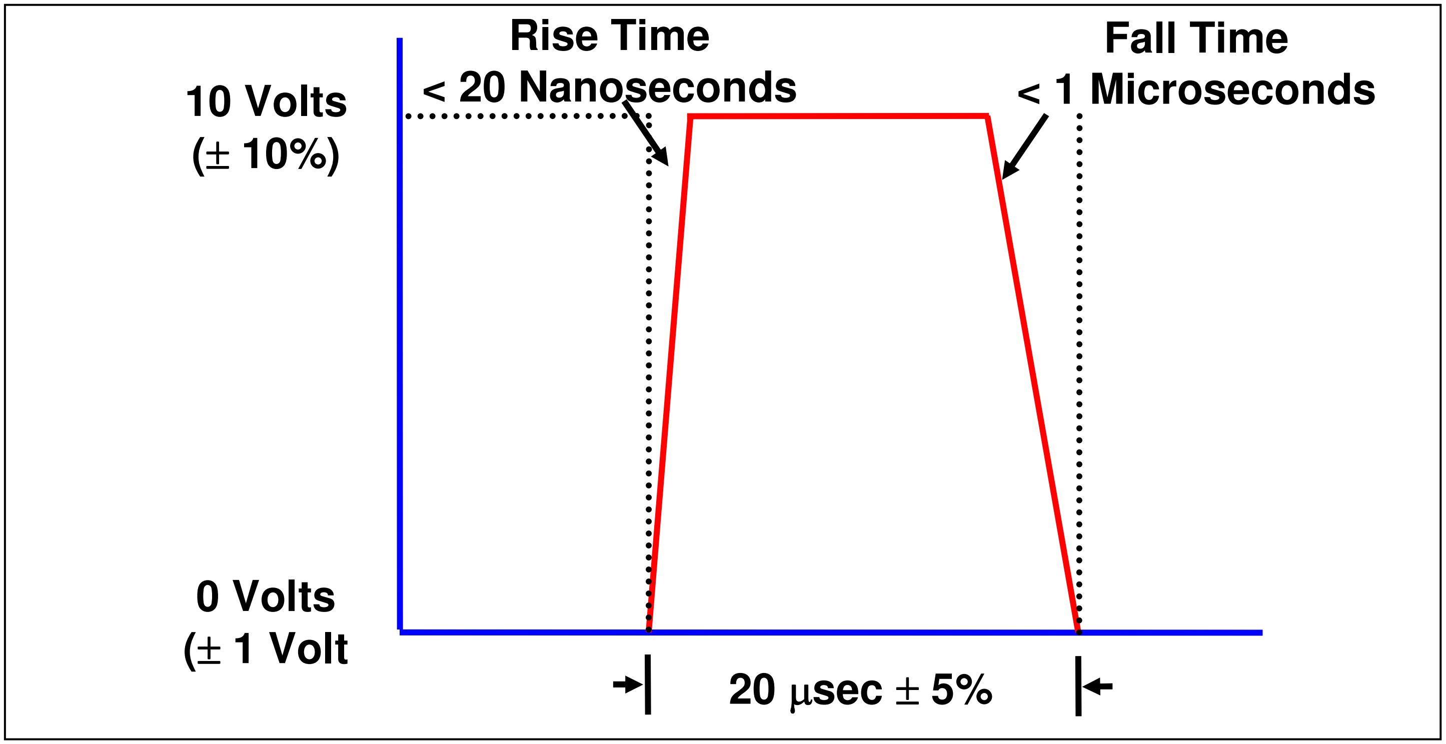
Figure 3. Format B: BCD time-of-year in days, hours, minutes and seconds; straight binary seconds-of-day plus optional control bits.

One Pulse-Per-Second

- The rising edge of a 1 Pulse-Per-Second (1PPS) signal is used to precisely define the beginning of the Second
- Standard 1PPS is defined by MIL-STD-188-115.
- 1PPS signal is equivalent to a frequency of 1Hz and can provide for accurate frequency as well. E.g., one can phase lock a 10MHz oscillator to a 1PPS signal from a GPS receiver to stabilize the oscillator in the medium and long term

One Pulse-Per-Second Timing Signal

(MIL-STD-188-115)



"The leading edge of the BCD code (negative going transitions after extended high level) shall coincide with the on-time (positive going transition) edge of the one pulse-per-second signal to within ±1 millisecond." See next page for the MIL-STD BCD code.

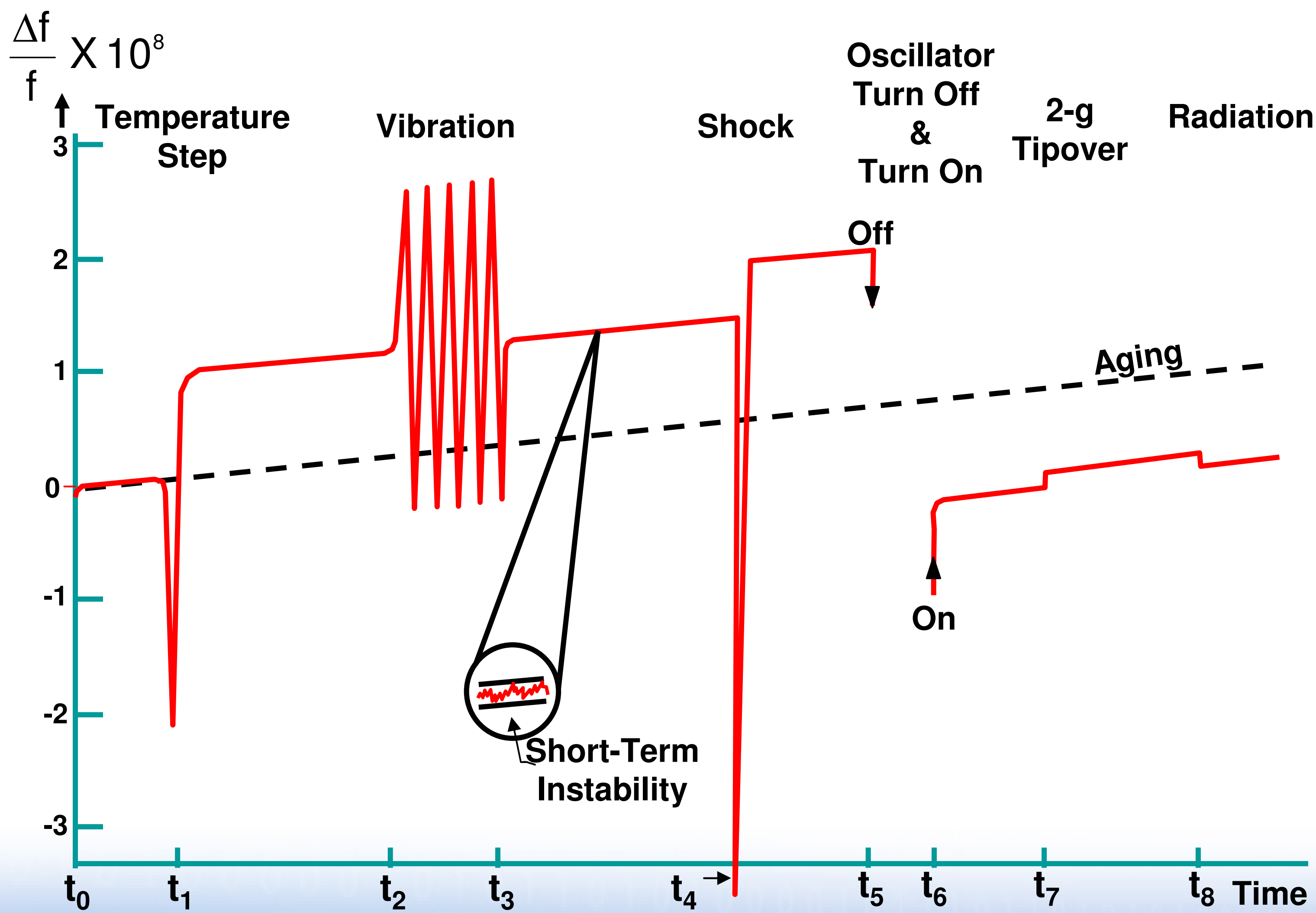
Influence of Environmental Conditions on Accuracy

Heartbeat of the Global Village!

Influences on Oscillator Frequency

- **Time**
  - Short term (noise)
  - Intermediate term (e.g., due to oven fluctuations)
  - Long term (aging)
- **Temperature**
  - Static frequency vs. temperature
  - Dynamic frequency vs. temperature (warmup, thermal shock)
  - Thermal history ("hysteresis," "retrace")
- **Acceleration**
  - Gravity (2g tipover)
  - Vibration
  - Acoustic noise
  - Shock
- **Ionizing radiation**
  - Steady state
  - Pulsed
  - Photons (X-rays, γ-rays)
  - Particles (neutrons, protons, electrons)
- **Other**
  - Power supply voltage
  - Atmospheric pressure (altitude)
  - Humidity
  - Load impedance
  - Magnetic field

Idealized Frequency Time Influence Behavior



Vibration-Induced Phase Excursion

The phase of a vibration modulated signal is

$$\varphi(t) = 2\pi f_0 t + \left(\frac{\Delta f}{f_v}\right)\sin(2\pi f_v t)$$

When the oscillator is subjected to a sinusoidal vibration, the peak phase excursion is

$$\Delta\varphi_{peak} = \frac{\Delta f}{f_v} = \frac{\left(\overline{\Gamma} \bullet \overline{A}\right) f_0}{f_v}$$

<u>Example</u>: if a 10 MHz, 1 x 10^{-9}/g oscillator is subjected to a 10 Hz sinusoidal vibration of amplitude 1g, the peak vibration-induced phase excursion is 1 x 10^{-3} radian. If this oscillator is used as the reference oscillator in a 10 GHz radar system, the peak phase excursion at 10GHz will be 1 radian. Such a large phase excursion can be catastrophic to the performance of many systems, such as those which employ phase locked loops (PLL) or phase shift keying (PSK).

Phase Noise Degradation Due to Vibration

- Data shown is for a 10 MHz, 2×10^{-9} per g oscillator
- Radar spec. shown is for a coherent radar (e.g., SOTAS)

Impacts on Radar Performance

- Lower probability of detection
  - Lower probability of identification
    - Shorter range
      - False targets